Math 300 Writing for Mathematics Spring 2003

Definitions, Procedures, and Explanations

There are three crucial modes of mathematical writing:

1. definition
2. procedure
3. explanation (theorem)

1 Definition

A definition is a *convention* which gives a name to a concept. Here are some examples.

The area of a region is the amount of space in the region. More precisely, the area of a region (expressed in square inches) is the number of squares 1 inch on a side (or fractions thereof) which cover the region.

The *diameter* of a (convex) region is the length of the longest line contained entirely in the region.

Consider a cross-section of a metal. Each crystal determines a region in the plane.

The *average area of a crystal* is the area of the cross-section divided by the number of crystals.

The *average diameter of a crystal* is the sum of the diameters of the crystals in a cross section divided by the number of crystals in the cross section.

2 Procedures

A procedure is a set of instructions for computing a function or property. Here are some partially worked out examples.

The following description is deliberately incomplete. *you must add more.* **In particular, the procedure for area should specify how to choose the rectangle in step 1, how the scale is used in step 2, and how partial crystals are treated in step 3. The procedure for 'diameter' or (1/AGI) should describe how any particular 'random' lines were chosen. 'I just drew four lines haphazardly ' or 'I took the lines on the handpoint' is a perfectly fine justification for this essay. Note that most of you were computing AGI not average diameter so your procedure will *not* be identical to mine.**

Consider a micrograph of a cross-section of a metal. The micrograph has a 'scale' attached to it.

To compute the average area of a crystal of this metal:

1. Choose a rectangle on the micrograph.
2. Determine the actual dimensions of the rectangle using the scale.
3. Count the number of crystals in the rectangle.
4. Divide the area in step 2 by the number of crystals in step 4.

Even more precisely, you might want to average over several rectangles.
Remember the procedures given here are incomplete.
To compute the average diameter of a crystal of this metal:

1. Choose randomly a number of lines on the micrograph. (They may have many different directions and lengths.)
2. Determine the actual length of the lines using the scale.
3. Count the number of crystals which intersect each line and divide it into the length of the line.
4. Average the numbers in step 3.

3 Explanation

An explanation gives reasons for believing a statement is true. The most formal kind of explanation is a proof. But in this course we have relied on more intuitive explanations. The first two essays required semiformal explanations of statements that were mathematically clear such as $\Sigma_0^\infty \frac{1}{2^n} = 2$. In this essay we considered the much less clearly defined statement below where the actual proof is not, I think, known to any of us. So we discussed the geometric ideas involved and gave some examples that suggest an answer. This is speculation; this is the way mathematics actually develops.

Statement: The AGI method provides a more precise indicator of the size of a crystal than the average area method.

Here is *a* possible summary of the argument. Many of you have provided different and adequate summaries. In the essay this summary might draw on a discussion of the differences between the AGI of square and rectangular crystals.

The AGI method provides a more precise indicator of the size of a crystal than the average area method. Roughly speaking, the average area of a sample will be proportional to the square of the average diameter. So if the average diameter is bigger, the average area will be bigger. But the average diameter distinguishes different shapes as we illustrated by the example of square and rectangular crystals.